local R_θ was varied by changing tunnel stagnation pressure at each of two total temperatures (700° and 1050°K). The resulting range of R_θ was 2,000 to 18,000 and T_w/T_{aw} was about 0.32 and 0.50. The skin-friction results were analyzed and reported by Hopkins et al.[3] who gave additional description of model and test conditions.

The Reynolds analogy factor is defined by the following relationship:

$$2C_h/C_f = V_e(\dot{q}_w/\tau_w)/(H_{aw}-H_w)$$

where

$$H_{aw} = H_e + rV_e^2/2$$

Consequently, the direct determination of $(2C_h/C_f)$ requires the simultaneous measurement of $\dot{q}_w$, τ_w, V_e, T_t, T_w and r, all of which were measured in the present investigation, except the recovery factor r, for which value of $r = 0.88$ was assumed. The use of other values of r for analyzing heat-transfer data is discussed in Ref. 2, however, $r = 0.88$ led to a more satisfactory correlation of heat transfer with skin-friction measurements.

Figure 1 presents the Reynolds analogy factor $(2C_h/C_f)$ as a function of R_θ. There is no distinct effect of Mach number and so all of the data are superimposed but identified by symbols. Maximum scatter is $\pm 9\%$, however, most of the results fall within 4% of $2C_h/C_f = 1.0$ over the large Reynolds number range presented $(2,000 < R_\theta < 18,000)$.

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Technical Comments

Comments on "Dynamics of an Explosive Reaction Center"

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ZAJAC and Oppenheim[1] claim to present "the first study of the gas dynamic effects of a reaction center in an explosive gas mixture." This claim is not valid because the authors failed to recognize the principal features of transient spherical and cylindrical flows. Zajac and Oppenheim postulate an impermeable interface between the surroundings and the reaction kernel in which the pressure and temperature are spatially uniform, but time dependent. Chemical reactions of hydrogen with oxygen are computed using 22 forward and backward steps whose rate constants are given by $K = AT^n \exp(-E/T)$ with various constants A, exponents n, and activation energies E for each reaction step. This rather complex chemical model is combined with the ideal gas law in which pressure and temperature are determined from the coupled solutions for uniformly reacting kernel and the ambient gas flow. In effect then, Ref. 1 considers a variation of the classical piston problem in which the expansion is implicitly determined by chemical reactions, rather than specified explicitly.

The expanding kernel generates a primary shock wave (considered by Ref. 1) and also a secondary shock wave (neglected by Ref. 1) which runs into the kernel, causes an order of magnitude spatial variation of temperature and pressure, and thus nullifies the spatial uniformity assumption of Ref. 1 on which all following calculations are based. The complex chemical and characteristics calculations of Ref. 1 refer to a fictitious system which cannot be found in reality.

It is well known (e.g., Stanyukovich,[2] Brode,[3] Friedman,[4] and Glass[5]) that spherical and cylindrical expansions of gases generate an inward-bound expansion fan, followed by a secondary shock wave which points away from the primary shock wave generated by the motion of the contact surface. Essentially, the overexpansion caused by the rarefaction fan and the volume change must be corrected by a shock wave to satisfy the condition of continuous pressure across the contact surface. Initially, the secondary shock wave is weak, but as it approaches the origin, it accelerates and theoretically goes to infinite strength at the origin.[3] An excellent discussion of spherical flows is found in Chapter VI-C of Courant and Friedrichs,[6] where it is pointed out that in the limiting case of strong converging shock waves (Guderley's solution, p. 432), the pressure behind the shock front reflected from the origin is 26 times the pressure behind the incident front. Upon reflection from the origin, the secondary shock wave moves outward and interacts with the interface, thus causing damped oscillations of the system.[2] In a pioneering analytical study of expanding gas spheres, Brode[3] pointed out that the repeated reflections of the secondary shock waves tend to concentrate the energy near the origin in a manner similar to that found in a point source solution. Analyses of such solutions in standard texts (e.g., Sedov[7]) show that pressures and temperatures theoretically go to infinity at the origin.

No exact calculations of the problems considered by Zajac and Oppenheim are presently available for direct comparison with their results. Nevertheless, errors may be assessed using the order of magnitude estimates of the spatial variations of temperature and pressure variations due to the disregarded gas dynamics phenomena. The theoretical limits of infinite temperatures at the origin are not realistic for estimating purposes and it is more appropriate to use existing related experimental data. Glass[5] discusses experimental and theoretical studies of expanding helium and air spheres with initial pressure ratios of 18 and 22, respectively. The surrounding air was initially at the same temperature as the sphere gases. Using the artificial viscosity method like the one employed by Brode, Glass showed that, initially, the pressures and temperatures at the origin decrease by an order of magnitude and subsequently increase by two orders of magnitude when the secondary shock wave is reflected. These results were supported by the experimental data. The pressure ratios in Ref. 5 were higher than those considered by Zajac and Oppenheim (highest pressure ratio 12), but in view of the known theoretical limits, the results obtained by Glass are indicative of the magnitudes of temperature and pressure changes to be expected when the full gas dynamics problem is considered. For a 0.1 mm kernel Figs. 8 and 9 of Ref. 1 show that the temperature rise time and the power pulse width are about 0.1 μsec. With the acoustic velocity of 1 mm/μsec, given in Ref. 1, the time for the initial rarefaction fan to reach the origin is also about 0.1 μsec. Thus, in roughly the characteristic time of the problem an order of magnitude variation of temperature is established

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along the kernel radius. The secondary shock wave is initially so weak that it is swept outward by the expanding flow but it gains strength and, as shown by Friedman,[4] at about the time that the rarefaction fan reaches the origin the secondary shock wave is moving inward, even in stationary coordinates. This shock wave is initially quite weak so that it reaches the origin about 0.1 μsec after the rarefaction fan is reflected. The reflected shock front now establishes an order of magnitude variation of temperature along the radius, but this time the maximum is at the origin. Successive reflections and interactions with the contact surface reverse the temperature gradients along the kernel radius with a period of about twice the characteristic time of the problem. The assumption of uniform kernel temperature cannot be justified. The power law and exponential expression for the temperature dependence of the reaction rates magnify the order of magnitude errors introduced by the incorrect gas dynamics model assumed by Ref. 1 and thus, the whole calculation is meaningless.

A further point which, although relatively minor compared to the fundamental objections raised above, nevertheless deserves mention in order to view the work of Zajac and Oppenheim in the proper perspective. In the calculations of the flowfield ahead of the expanding spherical kernels, Ref. 1 did not predict the existence of the primary shock wave at the head of the disturbed flow. The absence of the shock wave ahead of the accelerating contact surface was dismissed with the statement that "Shock waves were generated only in the case of plane and cylindrical flows, the increase of the cross-sectional area in the spherical case having been evidently too large for this purpose." The inability to calculate a shock wave suggests numerical difficulties; moreover, the accompanying statement cannot be accepted since it contradicts directly the discussion on p. 428 of Courant and Friedrichs, who demonstrate clearly that even a uniformly expanding sphere generates a shock wave.

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Reply by Authors to A. Wortman

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EVIDENTLY, the commentator fell a victim to many misconceptions. Nonetheless the authors appreciate his comments for they bring up a number of interesting points that might have been indeed missed by a casual reader.

The basic feature of our studies has been described by him quite correctly at the end of the first paragraph although at the start he accused us of "failing to recognize the principal features of transient... flows." He then went on to describe the well-known solutions of the so-called spherical or cylindrical shock tube problem where, as it was first shown by Wecken in 1950 (Ref. 1), a secondary imploding shock wave is formed. As succinctly phrased by Friedman (Wortman's Ref. 4), "the physical reason for the secondary shock formation is that the high-pressure gas, upon passing through a spherical rarefaction wave, must expand to lower pressures than those reached through an equivalent one-dimensional expansion." Finally he pointed out, with reference to the text of Courant and Friedrichs, that even a uniformly expanding sphere generates a shock wave—a fact that has been well established since the first classical paper in blast wave theory written by G. I. Taylor in 1939 and published in 1946 (Ref. 2).

It is the lack of the evidence in our results of the existence of the secondary shock and, in the spherical case, of even the primary shock that forms the major issue in the critique of the commentator. What he evidently overlooked was the fact that, in all the cases to which he referred, the time for the formation of the shock was negligibly small in comparison to the scale of the phenomenon under study. The existence of the shock has to be, in fact, postulated in these cases, in order to satisfy the dynamic compatibility conditions for the whole flowfield.

In contrast to this, our study is concerned with the phenomena occurring within the same order of magnitude in time as that of shock wave formation, belonging therefore to a different class of problems, namely that which, to quote another classical reference, has been studied in the 1940's by Friedrichs[3] who concluded significantly: "Thus it is clear that, in detail, the formation of the shock, at least in its initial stages, depends in a very sensitive way on the motion of the piston which produces the wave" (p. 238 of Ref. 3).

Our studies were then of a pioneering character in one major respect, namely that, as Wortman recognized, they provided information on the motion of the "piston" associated with a given kinetic scheme of exothermic chemical reactions. In order to be able to take properly into account the chemical kinetic and the thermodynamic properties of the reacting system, we had to

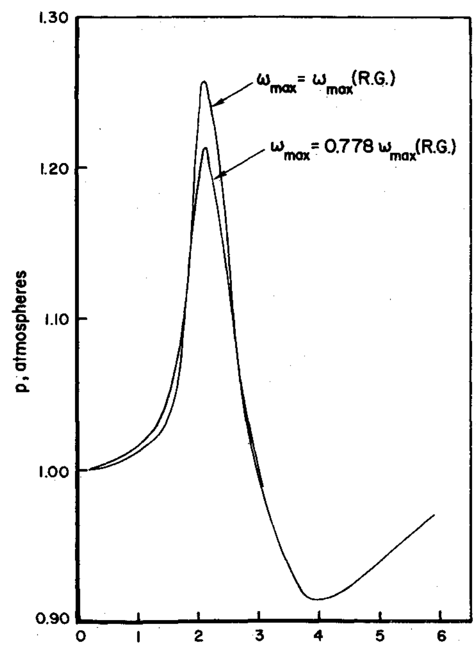

Fig. 1 Pressure pulse computed for a bulk expansion model in perfect gases with constant specific heats for a given exothermic power pulse.

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